Fuzzy bags and Wilson lines

The pressure, near T_c, as a "fuzzy" bag

1. Helsinki program of resumming perturbation theory

   Non-perturbative terms in the pressure

The sQGP from Wilson lines in *weak* coupling

2. (Some) large gauge transformations.

   Interfaces, Z(N) and U(1), and their uses.

3. The electric field in terms of Wilson lines.

4. Confinement as an (adjoint) Higgs effect

Helsinki Program

Match original theory in 4D, to effective theory in 3D, for r > 1/T

$$\mathcal{L}^{eff} = \frac{1}{2}\,\mathrm{tr}\,G_{ij}^2 \;+\; \mathrm{tr}\,|D_i A_0|^2 + \; m_D^2\,\mathrm{tr}\,A_0^2 \;+\; \kappa\,\mathrm{tr}\,A_0^4$$

$m_{Debye}^2 \sim g^2 T^2$, $\kappa \sim g^4$, series in g^2 .

(First step in three: then resum m_{Debye} , $m_{magnetic}$)

"Optimal" resummation of perturbation theory: valid for *small* A_0

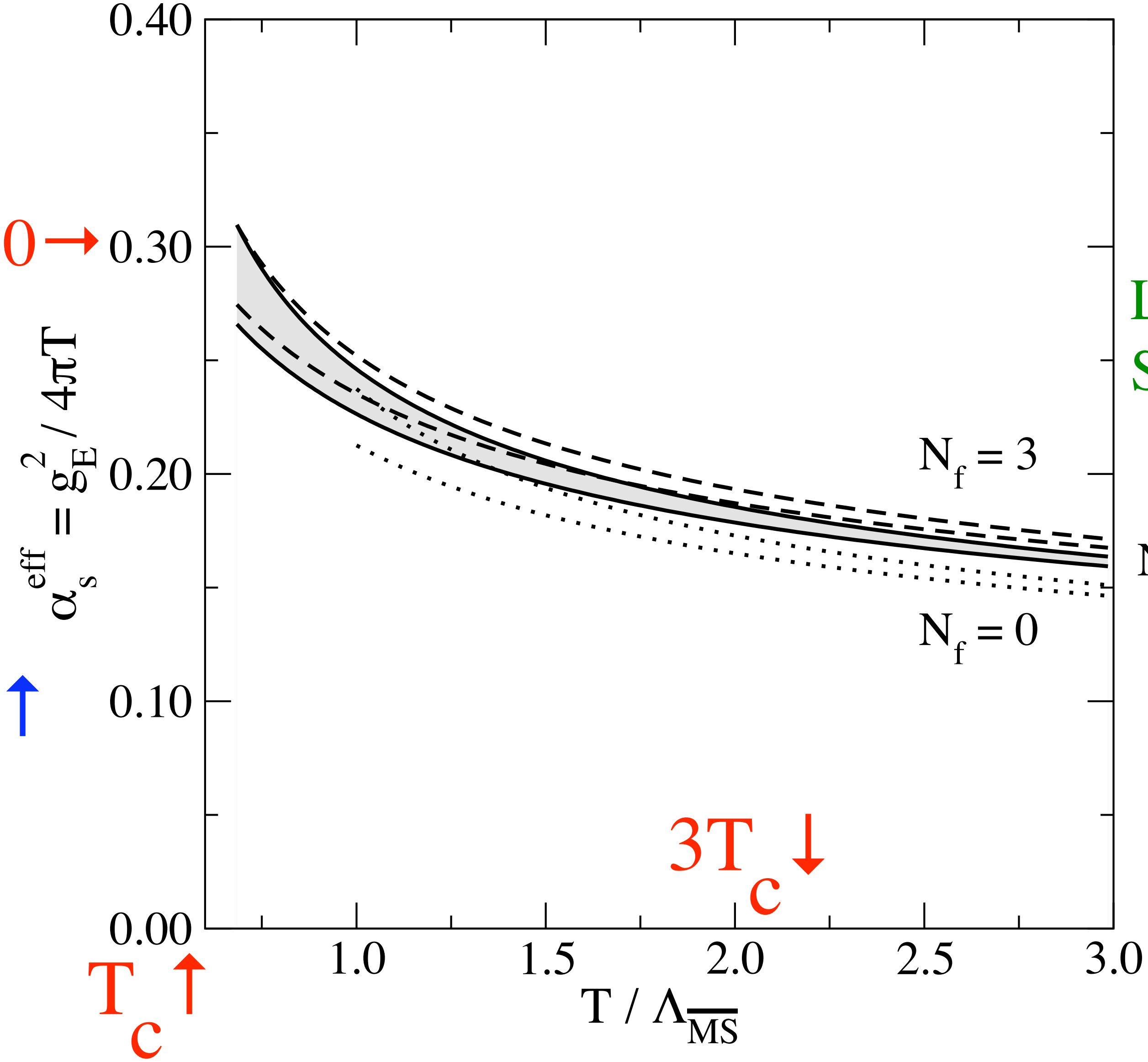
How does α_s^{eff} run? Braaten & Nieto '96: $\alpha_s^{eff}(2\,\pi\,T)$?

Even better! Laine & Schröder '05: 2-loop calc. $\Rightarrow$ $\alpha_s^{eff}(9\,T)$!

$T_c \sim 175$ MeV: $9\,T_c \sim 1.6$ GeV, $\alpha_s^{eff}(9\,T_c) \sim 0.28$

$9\,(3\,T_c) \sim 4.8$ GeV : T_c to $\sim 3\,T_c$ *not* (so) strong coupling

α_s^{eff} is *not* so big, even at T_c



0.30 →

Laine & Schröder ’05

$\alpha_s^{eff}(T) \uparrow$

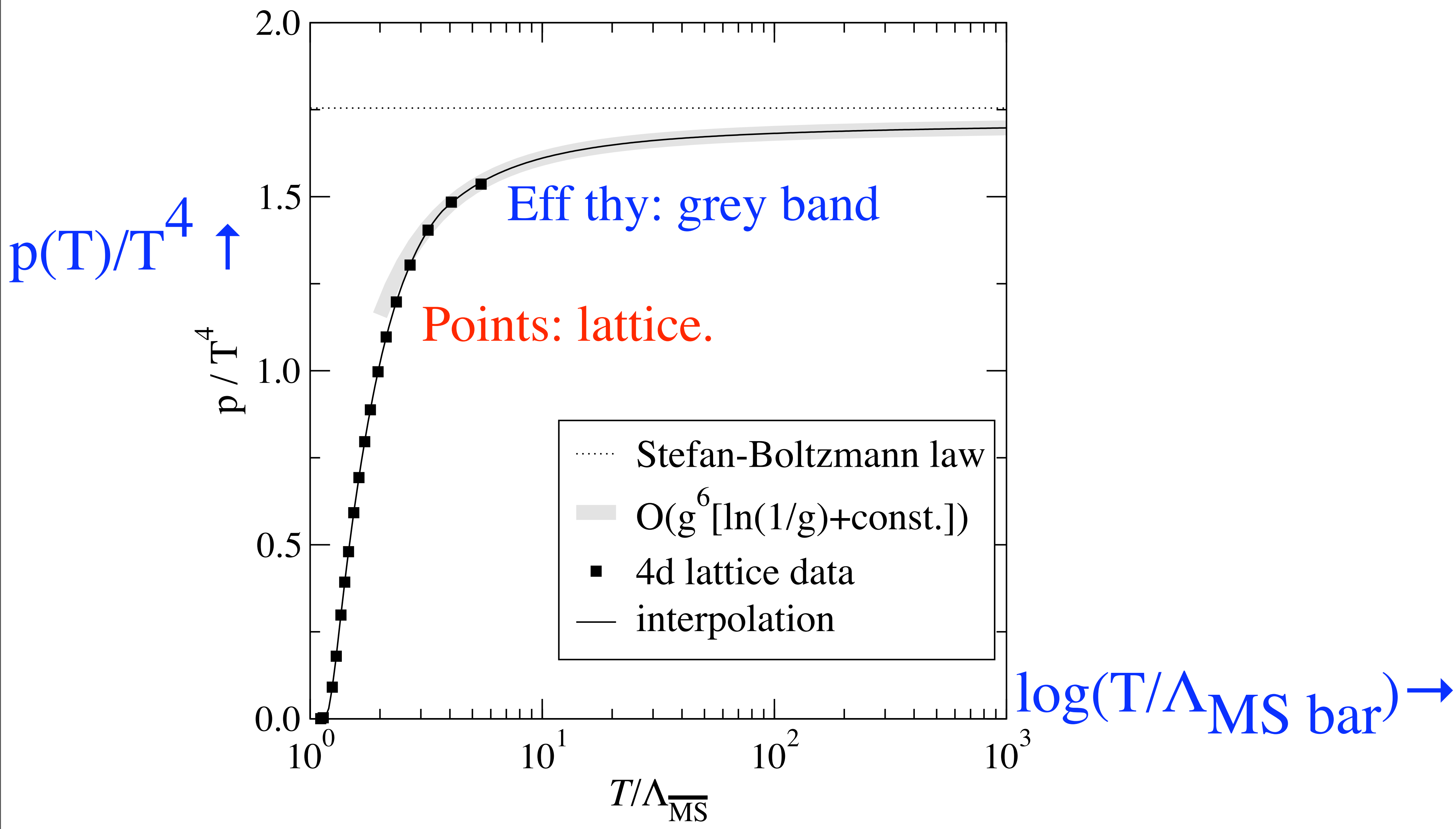
$T_c \uparrow$

$T/T_c \rightarrow$

$\alpha_s^{eff}(c\,T)$: $c \sim 2\,\pi \rightarrow 9$. Might have been $2\,\pi \rightarrow 2$.

If so, then strong coupling below 3 T_c. *Not* what happens.

Pressure: effective theory *fails* below ~ 3 T_c



If α_s^{eff} is not so big, why *doesn't* effective thy work for the pressure?

Old story: Lattice pure SU(3) glue, $(e-3p)/T^4$

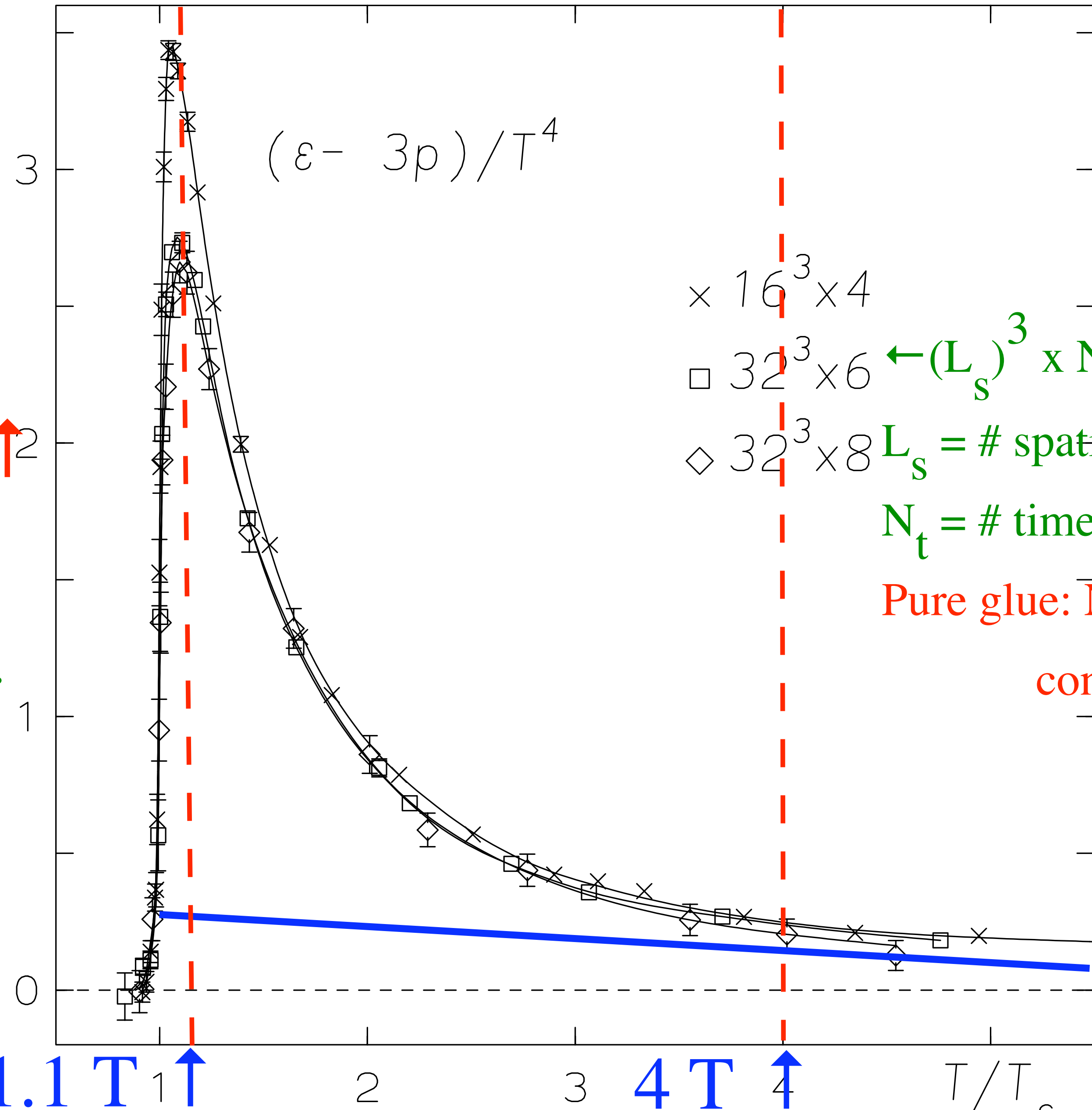
e = energy

p = pressure

$(e-3p)/T^4$ ↑

Trace of energy - mom. tensor/T^4 ↑

Bielefeld, lat/9602007



1.1 T_c ↑

4 T_c ↑

← $(L_s)^3$ x N_t:

L_s = # spatial lattice spacings

N_t = # time steps

Pure glue: N_t = 6,8 *close* to continuum limit

← Perturbative cont. ~ α_s^2

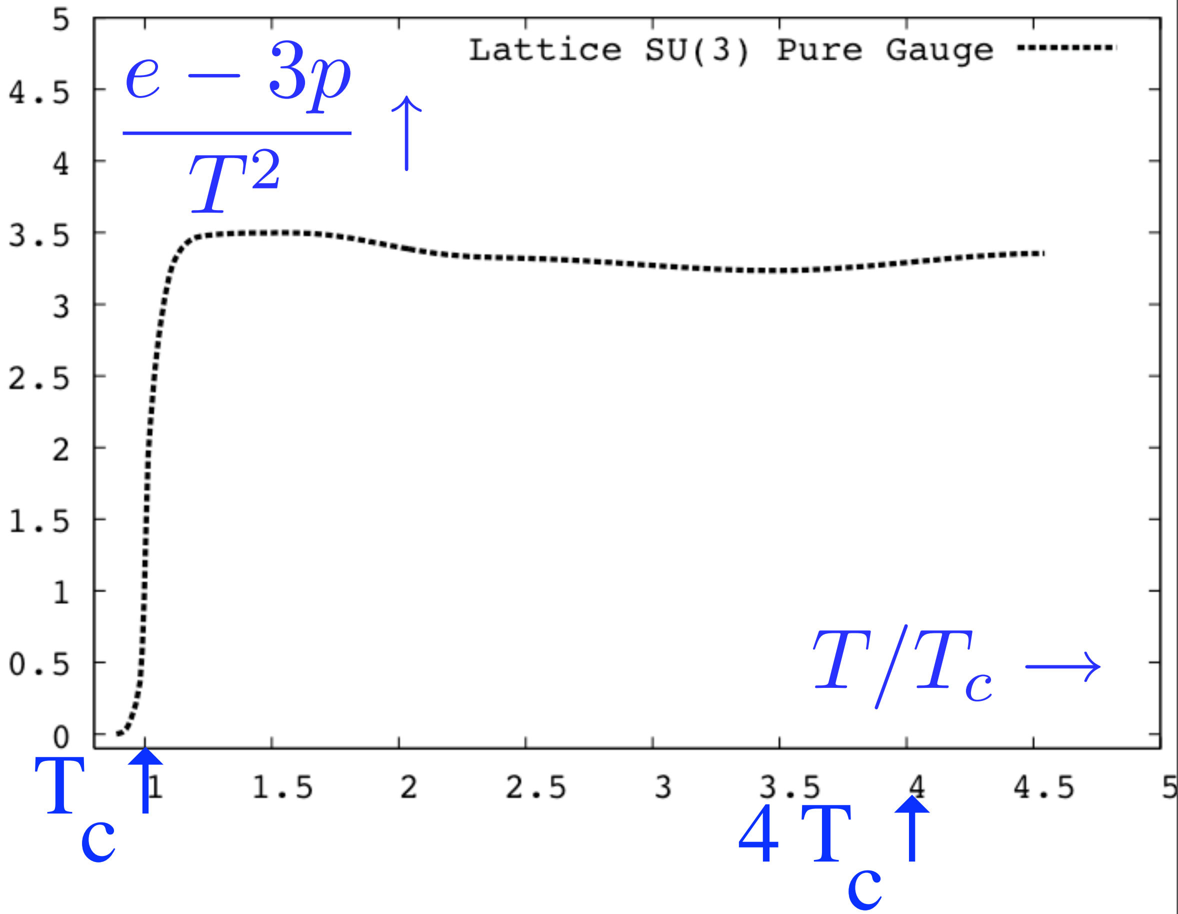
"Fuzzy" bags

Now plot (e-3p)/T^4 *times* T^2:
constant from 1.1 T_c to 4 T_c!

So p(T) = sum of *only* T^4, T^2
Since p(T_c) is small, for *pure* glue:

$$p(T) \approx f_{pert}(T^4 - T_c^2\,T^2)$$

f_{pert} ~ constant, T: 1.1 T_c to 4.0 T_c



With dynamical quarks: perhaps for T> T_c, pressure a series in 1/T^2:

$$p(T) = f_{pert}\,T^4 - B_{fuzzy}\,T^2 - B_{MIT} + \ldots$$

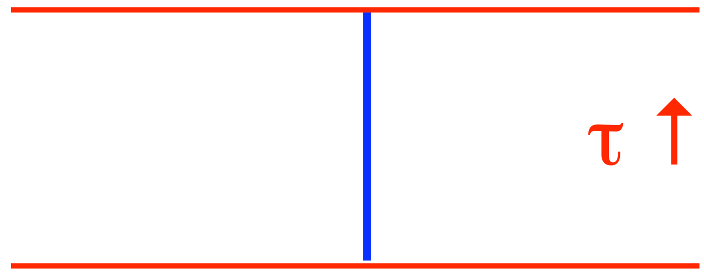
B_{fuzzy} "fuzzy" bag constant: dominates MIT bag constant, B_{MIT}, away from T_c
Maybe: only perturbative terms contribute to $f_{pert}(g^2)$: works down to T_c ?
Perturbation theory fails because of *non*-perturbative terms, powers in 1/T^2

Effective theory near T_c

Could use eff. thy. of *local* quasiparticles...

Or use (natural) *non*local variable, thermal Wilson line. Start with *straight* lines:

$$\mathbf{L}(x) = P\, e^{ig\int_0^{1/T} A_0(x,\tau)\, d\tau}$$



Under gauge transformations, $\mathbf{L}(x) \to \Omega(x, 1/T)^\dagger\, \mathbf{L}(x)\, \Omega(x, 0)$

For *periodic* $\Omega(\tau)$, traces are gauge invariant.

Polyakov loop: measures fraction of deconfinement. $\ell(x) = \mathrm{tr}\, \mathbf{L}/3$

Can extract renormalized Polyakov loop from lattice, after removing lattice "mass" renormalization. (Kaczmarek + ...'02....Orginos et al '03).

Perturbative regime: complete deconfinement. Loop near one, g A_0/T small.

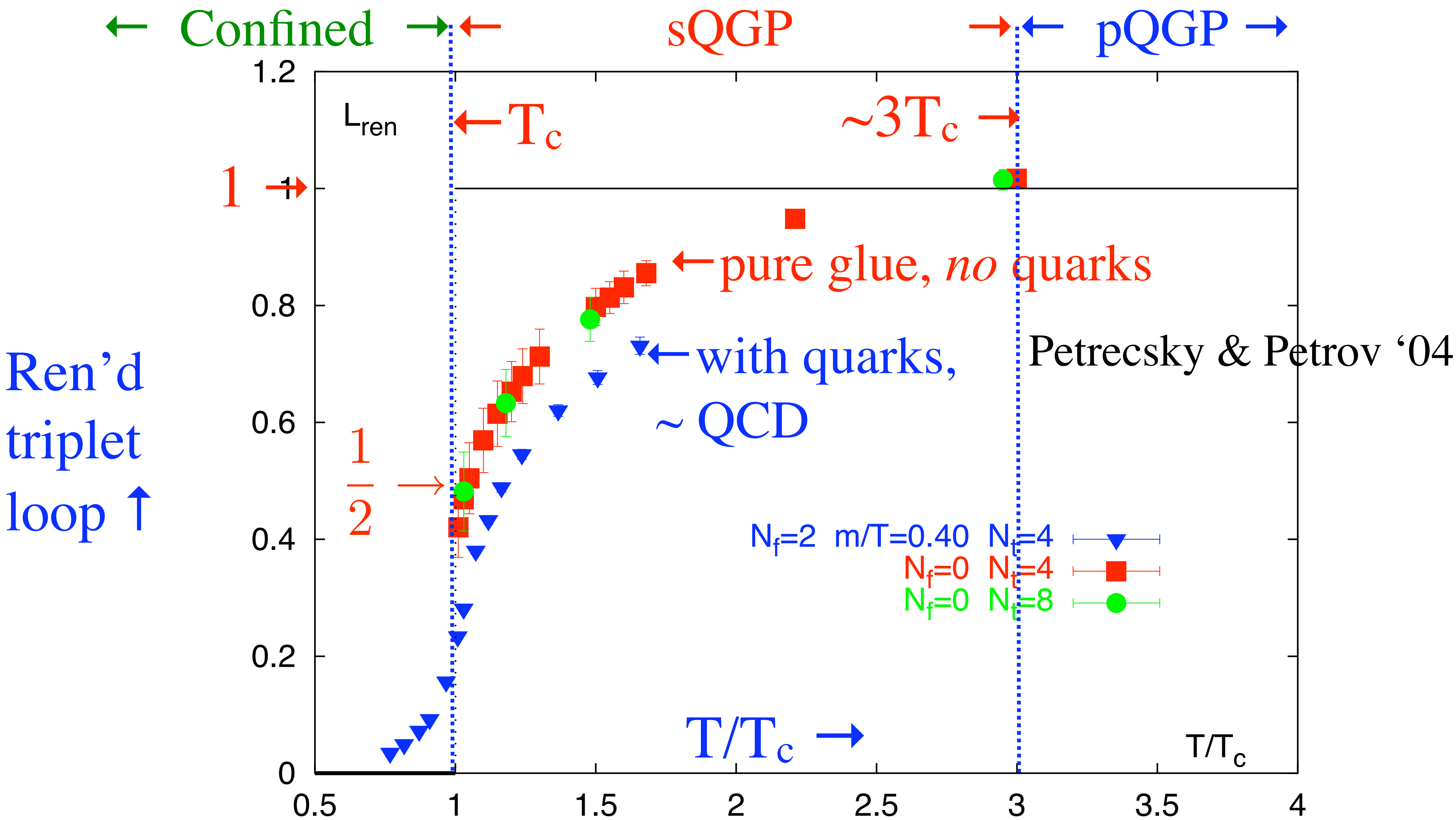
Non-perturbative regime: *partial* deconfinement. Loop < 1, so g A_0/T *large*.

“sQGP”: *partially* deconfined

From ren.’d Polyakov loop on lattice: $T > 3\ T_c$: loop ~ 1, ~ perturbative QGP

$T_c \rightarrow 3\ T_c$: loop < 1, *partial* deconfinement, “sQGP”

Need effective theory for *large* A_0



Effective theory for large A_0

Symmetries? Certainly, invariance under static gauge transf.'s.

Plus: "large" gauge transformations - spatially constant, time *de*pendent. For SU(N):

$$U_c(\tau) = e^{2\pi i\,\tau T\,t_N/N} \;\;,\;\; t_N = \begin{pmatrix} \mathbf{1}_{N-1} & 0 \\ 0 & -(N-1) \end{pmatrix}$$

This $U_c(\tau)$ is *only* valid c/o quarks: $U_c(1/T) = \exp(2\,\pi\,i/N)\,U_c(0)$

Shows center symmetry of pure SU(N) glue: a global Z(N) symmetry

With quarks? Consider $U_c(\tau)$ to N^{th} power! $U_c(1/T)^N = \exp(2\,\pi\,i) U_c(0)^N = \mathbf{1}$.

All theories must respect invariance under such *strictly* periodic gauge transf.'s.

For any gauge group, with any matter fields.

With center symmetry, or not. Even for QED.

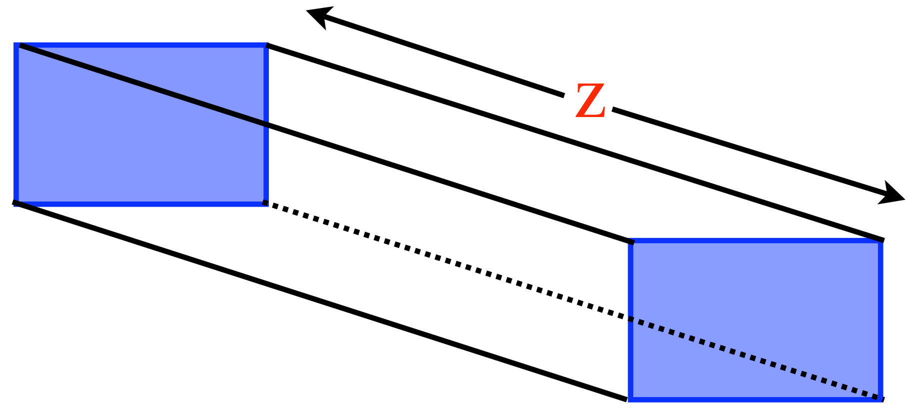
Strictly periodic, but large gauge transf.'s place nontrivial constraints on a *non*abelian effective theory.

Z(N) interfaces

One way to probe large A_0: Z(N) interface related to gauge transformation, $U_c(\tau)$

Take a long box:

$$\langle L \rangle = \mathbf{1}$$



$$A_0 = \frac{2\pi T}{gN}\, q(z)\, t_N$$

$$\langle L \rangle = \mathrm{e}^{2\pi i/N}\, \mathbf{1}$$

Take $A_0 \sim t_N$, times "coordinate" q(z).

Even at large A_0, the (original) electric field is abelian: $E_i^{4D} \sim \partial_i A_0 \sim dq/dz$.

L_{eff} = classical + 1 loop potential, for *constant* A_0

$$\mathcal{L}_{eff} = \mathrm{tr}\, E_i^2/2 + V_{1\,loop}(A_0) \sim \#(1/g^2(dq/dz)^2 + q^2(1-q)^2)$$

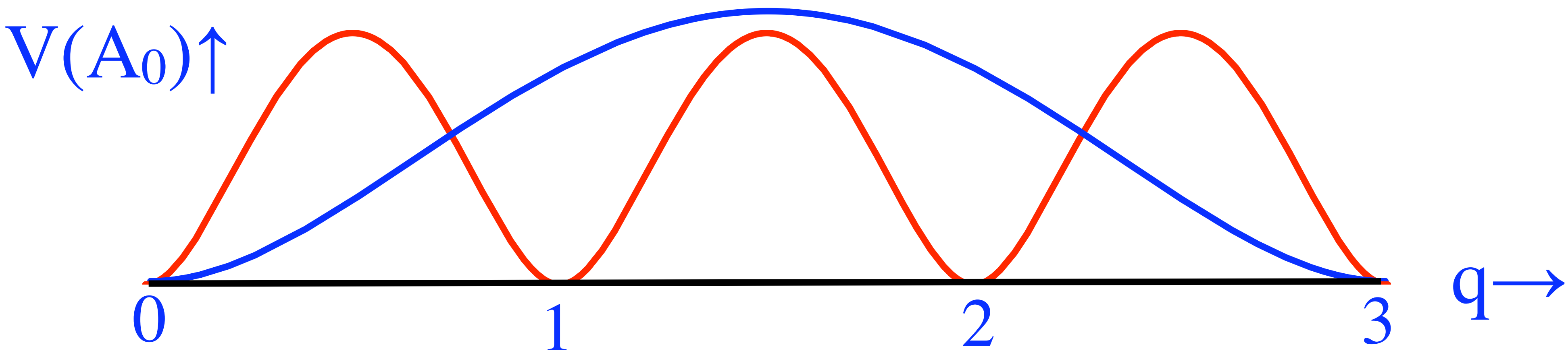
Usual tunneling problem: action ~ transverse area $\times$ # $T^2/(3\sqrt{g^2})$

Interface "fat": width ~ 1/(gT), so can use derivative expansion.

= $4\,\pi^2\,(N-1)T^2/\sqrt{(3N)}$. Compute semiclassically, now $(\sqrt{g^2})^3 \times$ # Korthals Altes

U(1) interfaces

What if no center symmetry? QCD: SU(3) with dynamical quarks, G(2)...
Use "U(1)" interface for *strictly* periodic gauge transf. In QCD, $U_c(\tau)^3$



Red: potential for constant A_0 from SU(3) gluons
For integer q, <L> = $\exp(2 \pi i q/3)$ **1**. q = 0, 1, 2 are degenerate Z(3) vacua.

Blue: potential from quarks. Potential at q = 1, 2 ≠ q = 0 , 3: *no* Z(3) symmetry
Still have U(1) interface: <L>: **1** → **1**, but q(z): 0 → 3.

Use U(1) interfaces to probe large A_0 . Properties gauge invariant, physical.
Associated with U(1) topology in maximal torus.

Effective electric field?

Want 3D effective thy. for large $A_0 \sim T/g$.

Valid for $r > 1/T$, so A_0 varies slowly in space, momenta $p < T$.

Original electric field $E_i^{4D} = D_i A_0 - \partial_0 A_i$. So $E_i^{3D} = D_i A_0$?

For large gauge transf. $U_c(\tau)^N = \exp(2\pi i T \tau t_N)$:

$$A_0^{diag} \to A_0^{diag} + \frac{2\pi T}{g} t_N \; , \; A_i \to \frac{1}{-ig} \, \Omega_c^\dagger(\tau) A_i \, \Omega(\tau)$$

Constant shift in A_0 , time *de*pendent rotation of A_i .

$D_i A_0 = (\partial_i - i g [A_i ,) A_0$ *not* invariant if $[A_i , t_N] \neq 0$.

Of course, E_i^{4D} invariant under $U_c(\tau)$.

$E_i^{3D} = D_i A_0$ at small A_0, but *not* at large A_0! Diakonov & Oswald '03, '04

Form E_i^{3D} from Wilson lines?

Electric field of Wilson lines

Wilson line SU(N) matrix, so diagonalize: $\mathbf{L}(x) = \Omega(x)^\dagger e^{\,i\lambda(x)}\,\Omega(x)$

Static gauge transf.'s: diagonal matrix λ invariant, Ω changes.

Strictly periodic $U_c(\tau)^N$: $\lambda_a \rightarrow \lambda_a + 2\pi \times$ integer: λ_a periodic. Of course!

Use just eigenvalues, $E_i^{3D} \sim \partial_i \lambda$? No, $E_i^{3D} \neq D_i A_0$ at small A_0

E_i^{3D} hermitean, so: $$E_i^{3D}(x) = \frac{T}{ig}\mathbf{L}^\dagger(x) D_i \mathbf{L}(x)(1 + c_1|\mathrm{tr}\mathbf{L}|^2 + \ldots)$$

Small A_0 OK, but does *not* fix c_1, c_2...

Large but *abelian* A_0, $A_i = 0$: if $E_i^{3D} = \partial_i A_0$, *must* have $c_1=c_2=\ldots=0$.

Necessary for interfaces to match at *leading* order. Beyond: c_1, c_2 ... $\sim g^2$.

In general, *infinite* number of terms enter.

Calculable perturbatively, match through interfaces, Z(N) or U(1).

$\mathcal{L}_{\rm eff}$ of Wilson lines at 0th order

To leading order, $$E_i^{3D} = \frac{T}{ig}\,\mathbf{L}^\dagger\, D_i\, \mathbf{L}$$

Gauge covariant "average" in time: $\mathbf{L}(\tau) = e^{ig\int_0^\tau A_o(\tau')d\tau'}$; $\mathbf{L} = \mathbf{L}(1/T)$

$$E_i^{3D}/T = \int_0^{1/T} d\tau\; \mathbf{L}(\tau)^\dagger\, \partial_i A_0(\tau)\, \mathbf{L}(\tau) - \mathbf{L}^\dagger[A_i, \mathbf{L}]$$

Math.'y: left invariant one form (Nair).

Lagrangian continuum form of Banks and Ukawa '83, on lattice: $$\mathcal{L}_{cl}^{eff} = \frac{1}{2}\,\mathrm{tr}\, G_{ij}^2 + \frac{T^2}{g^2}\,\mathrm{tr}\,|\mathbf{L}^\dagger D_i \mathbf{L}|^2$$

To 0th order, Lagrangian for SU(N) principal chiral field.

Non-renormalizable in 3D, but only effective theory for r > 1/T.

Instanton number in 4D = winding number of **L** in 3D

Linear model: Vuorinen & Yaffe '06 (Match by imposing extra symmetry)

Confinement & adjoint Higgs phase?

Diagonalize $L = \Omega^\dagger \, e^{i\lambda} \, \Omega$

Static gauge transf.'s U: $e^{i\lambda}$ invariant, Ω not: $\Omega \to \Omega\,\mathcal{U}$, $D_i \to \mathcal{U}^\dagger D_i\,\mathcal{U}$

Electric field term:

$$\mathrm{tr}\,|\mathbf{L}^\dagger D_i \mathbf{L}|^2 \;=\; \mathrm{tr}\,(\partial_i\lambda)^2 \;+\; \mathrm{tr}\,|[\Omega\, D_i\, \Omega^\dagger, \mathrm{e}^{i\lambda}]|^2$$

1st term same as abelian

2nd term gauge *in*variant coupling of electric & magnetic sectors

$\langle e^{i\lambda}\rangle = 1$: no Higgs phase. True in perturbation theory, order by order in g^2

If $\langle e^{i\lambda}\rangle \neq 1$, Higgs phase,

In weak coupling, diagonal gluons massless,
off diagonal massive (a,b = 1...N)

$$m^2_{ab} = g^2|\mathrm{e}^{i\lambda_a} - \mathrm{e}^{i\lambda_b}|^2$$

But for 3D theory, gluons couple *strongly*. Effects of Higgs phase?

N.B.: above 't Hooft's abelian projection for Wilson line.

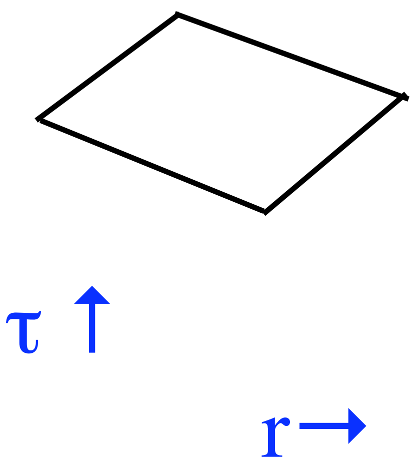
How to tell if adjoint Higgs phase?

No absolute, gauge invariant measure. Only differences qualitative.

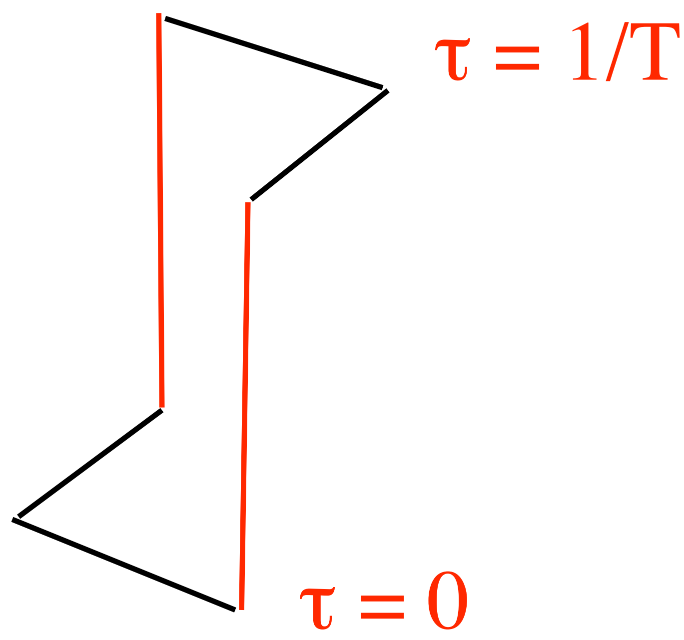
But: usually magnetic glueballs and Wilson line mix *very* little.
Higgs phase should *strongly* mix glueballs and Wilson line.

Maybe: measure magnetic glueballs from plaquettes “split” in time:

Usual spatial plaquette



“Split” spatial plaquette



Loop potential, perturbative & not.

U(N): constant **L**, 1 loop order:

$$\mathcal{L}^{eff}_{1\ loop} = -\frac{2\,T^4}{\pi^2}\sum_{m=1}^{\infty}\frac{1}{m^4}\,\left|\mathrm{tr}\,\mathbf{L}^m\right|^2 .$$

Perturbative vacuum $\langle e^{i\lambda}\rangle = 1$,
stable to leading order, to *any* finite order in g^2 .

Can compute corrections to effective Lagrangian at next to leading order, NLO.
At NNLO, ~ g^3 , need to resum m_{Debye} . Eventually, $m_{magnetic}$

SU(3) lattice: near T_c , pressure(T) ~ T^4 *and* ~T^2 .

To represent: add, *by hand:*

$$\mathcal{L}^{eff}_{non-pert.}(\mathbf{L}) = +\,B_f\,T^2\ \left|\mathrm{tr}\,\mathbf{L}\right|^2$$

$B_f \sim \#\, T_c^2$ "fuzzy" bag const. Non-pert., infinity of possible terms.

$B_f \neq 0 \Rightarrow \langle e^{i\lambda}\rangle \neq 1 \Rightarrow$ Higgs phase near T_c

Confinement in L_{eff}

SU(N), no quarks: in confined state, all Z(N) charged loops vanish:

$$\langle \mathrm{tr}\, \mathbf{L}^j_{\mathrm{conf}} \rangle = 0 \;,\; j = 1 \ldots (N-1)$$

Satisfied by "center symmetric" vacuum:

$$\mathbf{L}_{\mathrm{conf}} = \mathrm{diag}(1, z, z^2 \ldots z^{N-1}) \;,\; z = \mathrm{e}^{2\pi i/N} .$$

At finite N, perturbative pressure($\mathbf{L}_{\mathrm{conf}}$) *negative*. Not so good.

Large N: pressure($\mathbf{L}_{\mathrm{conf}}$) ~ 1, vs. ~ N^2 in deconfined phase.

At N=∞, center sym. state *can* represent confined vacuum.

$\mathbf{L}_{\mathrm{conf}}$ familiar from random matrix models:
completely *flat* eigenvalue distribution, from eigenvalue repulsion.

Where does eigenvalue repulsion arise *dynamically*?

Dynamical eigenvalue repulsion

Small volume: on *very* small sphere (R=radius, $g^2(R) < 1$ - Aharony et al.)

$L_{\rm eff}$ = random matrix model for constant mode. Measure gives eig. repulsion:

$$\mathcal{L}^{\rm eff}_{\rm Vandermonde} \sim - \sum_{a,b=1}^{N} \log(|e^{i\lambda_a} - e^{i\lambda_b}|^2)$$

Large volume: *no* sign of eigenvalue repulsion from perturbative loop potential.

Any term in measure regularization dependent.

Eig. repulsion arises, *naturally,* from adjoint Higgs phase: $m^2_{ab} \sim |\mathrm{e}^{i\lambda_a} - \mathrm{e}^{i\lambda_b}|^2$

One loop order in 3D:

$$\mathcal{L}^{\rm eff}_{\rm 1\ loop} \sim - \sum_{a,b=1}^{N} (g^2 |e^{i\lambda_a} - e^{i\lambda_b}|^2)^{3/2}$$

Two loop: $L_{\rm Vandermonde}{}^{\rm eff}$?

But: 3D theory strongly coupled: magnetic glueballs *heavy*, not light.

In $L_{\rm eff}$, confinement arises *uniquely* from (dynamical) eigenvalue repulsion.

Could study numerically. Field theory of "not so" random matrices.

Fuzzy bags and Wilson lines: credits

1. Helsinki program & renormalized loops

Resummation: Braaten & Nieto ’96. Andersen & Strickland ’04.
Kajantie, Laine, Rummukainen, & Schröder ’00, ’02, & ’03.
Giovannangeli ’05. Laine & Schröder ’05 & ’06. Di Renzo, Laine +... ’06
Renormalized loops: Kaczmarek, Karsch, Petreczky, & Zantow ’02 Dumitru, Hatta... below.
Petreczky & Petrov ’04. Gupta, Hubner, & Kaczmarek ’06

2. (Some) large gauge transformations

Large gauge transf.’s: Diakonov & Oswald ’03 & ’04. Megias, Ruiz Arriola, & Salcedo ’03.
Center symmetry, G(2): Holland, Minkowski, Pepe, & Wiese ’03. Pepe & Wiese ’06.
Z(N) interfaces: Korthals-Altes et al ’93, ’99, ’01, ’02, ’04

3. The electric field in terms of Wilson lines

Before: RDP ’00. Dumitru & RDP ‘00-’02. Dumitru, Hatta, Lenaghan, Orginos & RDP ’03
Dumitru, Lenaghan, & RDP ’04. Oswald & RDP ’05.
Linear model: Vuorinen & Yaffe ’06. Here, non-linear model: RDP ’06.
Lattice action: Banks & Ukawa ’83. Bialas, Morel, & Petersson ’04.

4. Confinement as an (adjoint) Higgs effect

Center symmetric vacuum: Weiss ’82. Karsch & Wyld ’86. Polchinski ’91. Schaden ’04.
Small sphere: Aharony, Marsano, Minwalla, Papadodimas, & Van Raamsdonk ’03 & ‘05